Simon-Gutowitz bidirectional traffic model revisited

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Abstract

The Simon-Gutowitz bidirectional traffic model (Phys. Rev. E 57, 2441 (1998)) is revisited in this letter. We found that passing cars get stuck with oncoming cars before returning to their home lanes. This provokes the occurrence of wide jams on both lanes. We have rectified the rules for lane changing. Then, the wide jams disappear and the revisited model can describe well the realistic bidirectional traffic.

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Road traffic congestion in reality is a complex phenomenon. It is the result of interactions between many road users. The formation of collective patterns of motion like traffic jams may be spontaneous or induced by the presence of bottlenecks e.g., on- and off-ramps, lane reductions, traffic lights or road works (see the reviews [1,2]). The cellular automata (CA) models are the most popular in the field of traffic flow modelling since they allow an effective implementation of real-time traffic computer-simulations (see the review [3,4]). In CA, time and space are discrete. The space is represented as a uniform lattice of cells with finite number of states, subject to a uniform set of rules, which drives the behavior of the system. These rules compute the state of a particular cell as a function of its previous state and the state of the neighboring cells. The most popular CA model for traffic flow on one-lane roadway is the NaSch model [5]. Despite its simplicity, the model is capable of capturing some essential features observed in realistic traffic like density waves or spontaneous formation of traffic jams. Different congested traffic states occur in other CA models: spontaneous jams caused by velocity fluctuations, synchronous phase, wide moving jams and stop-and-go phase [6].

A first step for describing the bidirectional traffic is given by Lee et al [7]. The authors have generalized the asymmetric exclusion model. In their model no passing is allowed. Instead oncoming traffic on the opposite lane reduces the hopping rates of the vehicles. Simon and Gutowitz [8] introduced a CA model for bidirectional two-lane traffic where vehicles move on two lanes with opposite directions. When a driver encounters a slower forward moving vehicle, a pass will be attempted. To do this, driver checks the density of vehicles in front that have to be passed, i.e. the local density. If this density is low enough the pass will be performed on condition of checking the safety criteria on the oncoming lane.

Up to now, Ref. [8] is the only paper to consider a CA model for bidirectional traffic flow. The Simon-Gutowitz model (SG model) is a probabilistic CA which consists of cars moving on two opposite lanes of L cells with periodic boundary conditions. Each cell is either empty, or occupied by just one car. In the model, there exist two types of cars: cars$^{[+]}$ moving on the lane$^{[+]}$ with positive direction and cars$^{[-]}$ moving on the lane$^{[-]}$ with negative direction (see figure 1a). We denote by x and v the position and the speed of a vehicle at time t respectively. The maximum speed of the cars is denoted by v_{max}. To distinguish between different interacting cars, several gaps and speeds are introduced. gap_{same} (gap_{opp}): the number of

unoccupied sites in front of a car on the same (opposite) lane. gap_{behind}: the number of unoccupied sites behind the car, on the opposite lane. v_{same} (v_{opp}): the speed of the car ahead on the same (opposite) lane. On the aim of making more compact the rules of the bidirectional model, several logical functions are introduced. **H**: true if the car is on its home lane; **oncoming**: true if car$^{[+]}$ and cars$^{[-]}$ are face-to-face on the same lane. **Space1**: true if $(gap_{same} < l_{pass})$ AND $(gap_{opp} > l_{security})$ AND $(gap_{behind} > l_{back})$. **Space2**: true if $(gap_{opp} > l_{security})$ AND $(gap_{behind} > l_{back})$. The parameters l_{pass}, l_{back} and $l_{security}$ are defined by the following. l_{pass}: if $gap_{same} < l_{pass}$ AND **H** then a pass may be attempted. l_{back}: the distance a driver looks back for obstacles on the passing lane. $l_{security}$: if $gap_{same} < l_{security}$ AND not(**H**) then the vehicle returns immediately to its home lane. D_L: local density: the fraction of the $l_{density} = 2v_{max} + 1 sites$ in front of the given vehicle which are occupied; D_{limit}: the maximum local density for a safe pass.
At each discrete time-step $t \rightarrow t+1$ the system update is performed in parallel for all cars according to the following subrules :
i. Lane changing rules:
1. IF (**H** AND **Space1** AND $(D_L \leq D_{limit})$ AND $(rand < p_{change})$) THEN change lane
2. IF (not(**H**) AND (($gap_{same} < l_{security}$) OR **Space2**)) THEN change lane.
The first condition concern vehicles on their home lane that want to change lane. When a driver encounters a slower forward moving vehicle, a pass is attempted. However, the pass will only be initiated if there is room far enough ahead on the passing lane, and the number of cars in front of the vehicle it would like to pass is small. Passing occurs randomly, even if all these conditions are met, the probability of changing lanes is denoted p_{change}. The second condition concerns vehicles in the midst of passing. They return to their home lane if forced to by an oncoming vehicle, or if there is space enough on the home lane that they can return without braking.
ii. Forward moving rules:
1. IF $(v \neq v_{max})$ THEN $v = v + 1$
2. IF((**oncoming**) AND $(gap_{same} \leq (2v_{max} - 1))$) THEN $v = gap_{same}/2$
3. IF ((not(**oncoming**)) AND $(v > gap_{same})$) THEN $v = gap_{same}$
4. IF (**H** AND $(v \geq 1)$ AND $(rand < p_{decel})$ AND not(**oncoming**)) THEN $v = v - 1$
5. IF (**H** AND (**oncoming**) AND $(v \geq 1)$) THEN $v = v - 1$.
If the vehicle is a car$^{[+]}$ then the vehicle moved forward according to: $x \leftarrow x + v$. But, if the vehicle is a car$^{[-]}$ then the vehicle moved backward accord-

ing to $x \leftarrow x - v$.
The rule (1) reflects the tendency of drivers to drive as fast as possible without exceeding the maximum speed limit. Rule (2) rapidly decelerates the vehicle if there is an oncoming vehicle too close. Rule (3) is intended to avoid collision between the vehicles of the same type. Rule (4) randomly decelerates the vehicle if it is on its home lane; if it is passing, it never decelerates randomly. Finally, rule (5) breaks the symmetry between the lanes, and thus prevents the emergence of a super jam, i.e., a jam which may occur when each of an adjacent pair of car$^{[+]}$ and car$^{[-]}$, one on each lane, tries to pass simultaneously.
In their paper, Simon and Gutowitz compared the traffic flow of the two-lanes bidirectional traffic with that of one-lane traffic. The improvement of the flow on one lane compared to the one-lane model depends on the density of vehicles on both lanes. Hence, for large densities on either both lanes there is little difference between the one-lane traffic model. When the density on the on-coming lane is small enough, then the flow on the home lane can be greater than in a one-lane model. Maximum improvement occurs near zero density on the on-coming lane. If the density on the home lane is small then the flow may be lower than in the corresponding one-lane model since when oncoming cars pass other oncoming cars they can impede traffic on the home lane.
In this letter, we reconsider the SG model where we are interested more especially in its congested patterns. It is clear that when the densities on the two lanes are all very low or all very high, the lanes will be effectively decoupled. However, in the case where the density of one lane is low and the one of the other lane is high, the interaction between lanes will become very important. In the SG model, there exist two different situations where a car in the midst of passing may return to its home lane. The first one is where the logical function **Space2** is satisfied. This describes the situation when the passing car returns to its home lane, before it is forced by an oncoming car. We find that this condition is hardly ever satisfied if the car density on the home lane is great enough. The second situation is when the passing car faces an oncoming car and thus will be forced to return to the home lane. This situation occurs if ($gap_{same} < l_{security}$). We found that this last situation is the one the more achieved in the SG model. Yet, almost all the passing cars provoke a head-on collision with on-coming cars and then stop until free space occurs on their home lane.
In our simulations we use the following values throughout: $l_{pass} = v$, $l_{back} =$

v_{max}, $l_{security} = 2v_{max} + 1$, $v_{max} = 5$, $p_{change} = 0.5$, and $p_{decel} = 0.3$. The system size is given by $L = 2000$.

Suppose that we have a small density of cars on lane$^{[+]}$ and a relatively high density on lane$^{[-]}$. Since free available spaces exist on lane$^{[+]}$, cars$^{[-]}$ pass other cars$^{[-]}$ and then impede traffic on lane$^{[+]}$. Cars$^{[-]}$ in the midst of passing will stop on lane$^{[+]}$ if they cannot return rapidly to their home lane. Indeed, all passing cars$^{[-]}$ get stuck with oncoming cars$^{[+]}$ before returning to their home lanes. Consequently, the cars$^{[+]}$ regroup into wide jams. Thereafter, passing cars$^{[-]}$ change to their home lane with keeping their speed equal to zero. As a result, the traffic of cars$^{[-]}$ on lane$^{[-]}$ will be delayed for a while. Obviously, the total delayed time will be important if the total number of passing cars$^{[-]}$ is important. Remark that, the number of passing cars will increase if the maximum local density for a safe pass (D_{limit}) increases. Congested patterns in SG model are illustrated in figure 2.

From our daily driving experiences in bidirectional traffic, we know that passing cars always return to their home lane before a head-on collision happens. That is why wide jams cannot exist in realistic bidirectional traffic without bottleneck. To prevent the occurrence of these wide jams it is necessary to allow returns of cars in the midst of passing before facing an oncoming car. Hence, we propose to rectify the lane changing rules in the SG model. Since the occurrence of a super jam is forbidden in the SG model, we don't need to require a free space ahead on the home lane for the passing car, i.e., $gap_{opp} > l_{security}$. Figure 1b can serve as an illustration. Hence, the logical function Space2 becomes:

Space2: true if $gap_{behind} > l_{back}$.

We define the revisited version of the SG model by considering the new version of the logical function **Space2** and by setting $D_{limit} = 1/l_{density}$. Congested patterns in the new version of the SG model are illustrated in figure 3. Hence, this figure shows clearly that wide jams disappeared. On lane$^{[+]}$, cars$^{[+]}$ move freely in spite of some obstructions caused by passing cars$^{[-]}$. On lane$^{[-]}$, we found density waves of cars$^{[-]}$ and a small number of passing cars$^{[+]}$ (see figure 3). Notice that, in contrast to the old version of the SG model, passing cars$^{[-]}$ always return to their home lane with non-vanishing speed. Besides, the head-on collisions between cars$^{[+]}$ and cars$^{[-]}$ occur rarely in the new version of the model.

Now we shall study the effect of varying the densities of cars on the traffic flow in the SG model. To do this, we fix the density of cars$^{[+]}$ and we vary the density of cars$^{[-]}$ ($\rho^{[-]}$). Suppose that the density of cars$^{[+]}$ is low

enough. When $\rho^{[-]}$ is very large, the two lanes are decoupled and the flow on lane$^{[+]}$ is identical to one-lane traffic flow. With decreasing $\rho^{[-]}$, the density of cars in front that have to be passed can be inferior to D_{limit} and then some cars$^{[-]}$ can pass on lane$^{[+]}$. We denote by $\rho_h^{[-]}$ the critical density of cars$^{[-]}$ above which the pass of cars$^{[-]}$ is forbidden. Thereafter, these passing cars$^{[-]}$ get stuck with oncoming cars$^{[+]}$ before returning to their home lane. This impedes traffic of cars$^{[+]}$ on lane$^{[+]}$. From figure 4, we observe that the flow of cars$^{[+]}$ decreases abruptly to a very small value. Indeed, the presence of only a few passing car$^{[-]}$ can create this abrupt decrease of the flow. With decreasing again $\rho^{[-]}$, the flow remains constant until the density reaches a critical density $\rho_l^{[-]}$. We find that $\rho_l^{[-]}$ is close to the critical density separating the free and congested states in one-lane traffic model. Below $\rho_l^{[-]}$, the traffic flow of cars$^{[+]}$ increases with $\rho^{[-]}$. This is not due only to the decrease of the number of passing cars$^{[-]}$ but also to the fact that their returns to home lane become more and more accessible.
It is clear that D_{limit} is a pertinent parameter in the SG model. If $\rho^{[-]}$ is very large, the fraction of the $l_{density}$ sites in front of a given car$^{[-]}$ which are occupied is almost equal to one. Therefore, if D_{limit} is small, the probability that a car$^{[+]}$ change lanes is zero. Hence, $\rho_h^{[-]}$ should decrease when one deceases D_{limit}. As figure 4 shows, the new version of the SG model presents a less important reduction of the traffic flow of cars$^{[+]}$ than those produced by the old version of the SG model. Furthermore, the critical density $\rho_h^{[-]}$ in the new version is lower than that in the old version of the SG model.
In figure 5 we plot the mean size of the longest cluster of cars$^{[+]}$ as a function of $\rho^{[-]}$. For low values of $\rho^{[-]}$, cars$^{[+]}$ regroup in clusters whose sizes increase with $\rho^{[-]}$. When $\rho^{[-]}$ exceeds $\rho_l^{[-]}$, the size of the largest cluster becomes great and vary almost constantly with $\rho^{[-]}$. This formation of wide jams leads to the maximal reduction of the traffic flow of cars$^{[+]}$. If $\rho^{[-]}$ exceeds $\rho_h^{[-]}$, the size of the largest cluster drops to a value equal to one. In this case, no interaction exist between lanes and the state of cars$^{[+]}$ will be free flow. Figure 5 illustrates clearly that the new version of the SG model do not exhibit wide jams but only a small clusters emerge in lane$^{[+]}$. These results are compatible with patterns shown in figure 3.
Suppose now that the density of cars$^{[+]}$ is relatively high. If $\rho^{[-]}$ is low, the flow of cars$^{[+]}$ will be greater than the flow of one-lane traffic model. As regards the effect of D_{limit}, we observe that when this last increases, the traffic of cars$^{[+]}$ is enhanced. Yet, the number of passing cars$^{[+]}$ should increase and

then will contribute enough to the flow of cars$^{[+]}$. With increasing $\rho^{[-]}$ the flow decreases. It becomes equal to the flow of one-lane traffic model when $\rho^{[-]}$ exceeds certain value $\rho_c^{[-]}$. The results are depicted in figure 6. In the revisited version of the SG model, the flow of cars$^{[+]}$ is slightly superior to the flow of one-lane traffic model.

In figure 7 we show the variation of the mean size of the longest cluster of cars$^{[+]}$ as a function of $\rho^{[-]}$. In the old version of the SG model, wide jams occur in lane$^{[+]}$ at low values of $\rho^{[-]}$. These wide jams disappear when $\rho^{[-]}$ exceeds $\rho_c^{[-]}$. In contrast, in the new version of the SG model, wide jams do not exist.

In summary, the SG model for bidirectionnel traffic flow is revisited. If the density of cars$^{[+]}$ is small and the one of cars$^{[-]}$ is high enough then wide jams occur in both lanes. The occurrence of these wide jams are due principally to the fact that almost all passing cars$^{[-]}$ get stuck with oncoming cars$^{[+]}$ before returning to their home lanes. The traffic flow of cars$^{[+]}$ is very small whereas the flow of cars$^{[-]}$ is greater than in the one-lane model. We have rectified the lane changing rules. As a result, the traffic flow of cars$^{[+]}$ is enhanced and the wide jams disappear. We believe that this revisited version of the SG model can describe well the realistic bidirectional traffic.

References

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Figures captions

Figure 1. Bidirectional model.
Figure 2. Congested patterns in SG model. On the left: lane$^{[+]}$: the black dots are cars$^{[+]}$ and the blue dots are passing cars$^{[-]}$. On the right: lane$^{[-]}$: the blue dots are cars$^{[-]}$ and the red dots are passing cars$^{[-]}$ which are returned to their home lane (their speeds are all equal to zero). The lattice size is $L = 400$, $D_{limit} = 2/l_{density}$ and the lane densities are : $\rho^{[+]} = 0.05$ and $\rho^{[-]} = 0.30$.
Figure 3. Congested patterns in the new version of the SG model. On the left: lane$^{[+]}$: the black dots are cars$^{[+]}$ and the blue dots are passing cars$^{[-]}$. On the right: lane$^{[-]}$: the blue dots are cars$^{[-]}$, the black dots are passing cars$^{[+]}$ and the red dots are passing cars$^{[-]}$ which are returned to their home lane (their speeds are all different from zero. The lattice size is $L = 400$ and the lane densities are : $\rho^{[+]} = 0.05$ and $\rho^{[-]} = 0.30$.
Figure 4. Flow of cars$^{[+]}$ as a function of the density of cars$^{[-]}$. The density of cars$^{[+]}$ is low enough ($\rho^{[+]} = 0.05$).
Figure 5. Mean size of the longuest cluster of cars$^{[+]}$ as a function of the density of cars$^{[-]}$. The density of cars$^{[+]}$ is low enough ($\rho^{[+]} = 0.05$).
Figure 6. Flow of cars$^{[+]}$ as a function of the density of cars$^{[-]}$. The density of cars$^{[+]}$ is high enough ($\rho^{[+]} = 0.30$).
Figure 7. Mean size of the longuest cluster of cars$^{[+]}$ as a function of the density of cars$^{[-]}$. The density of cars$^{[+]}$ is high enough ($\rho^{[+]} = 0.30$).

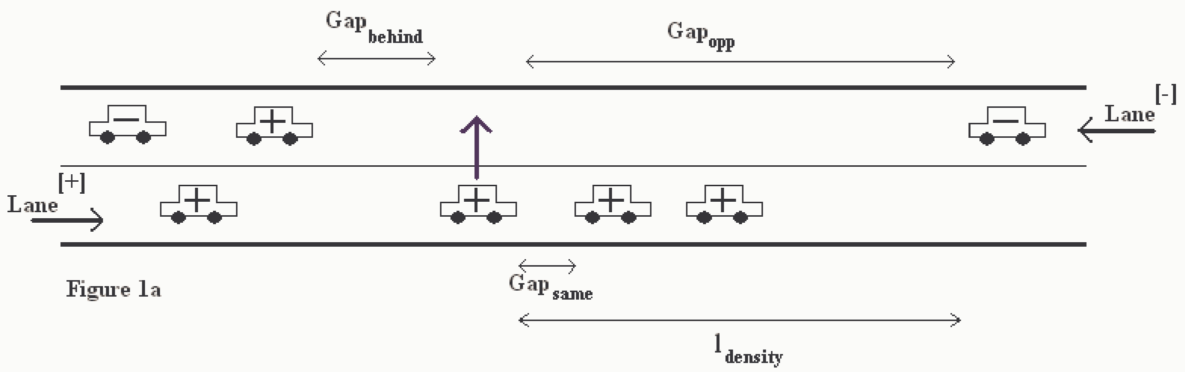


Figure 1a

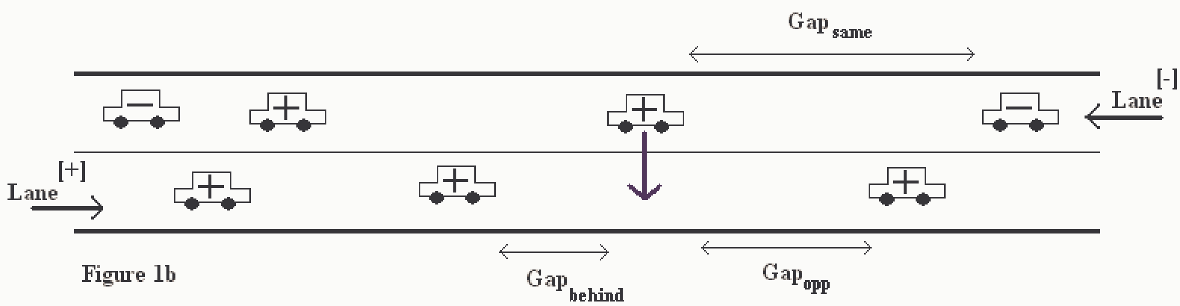
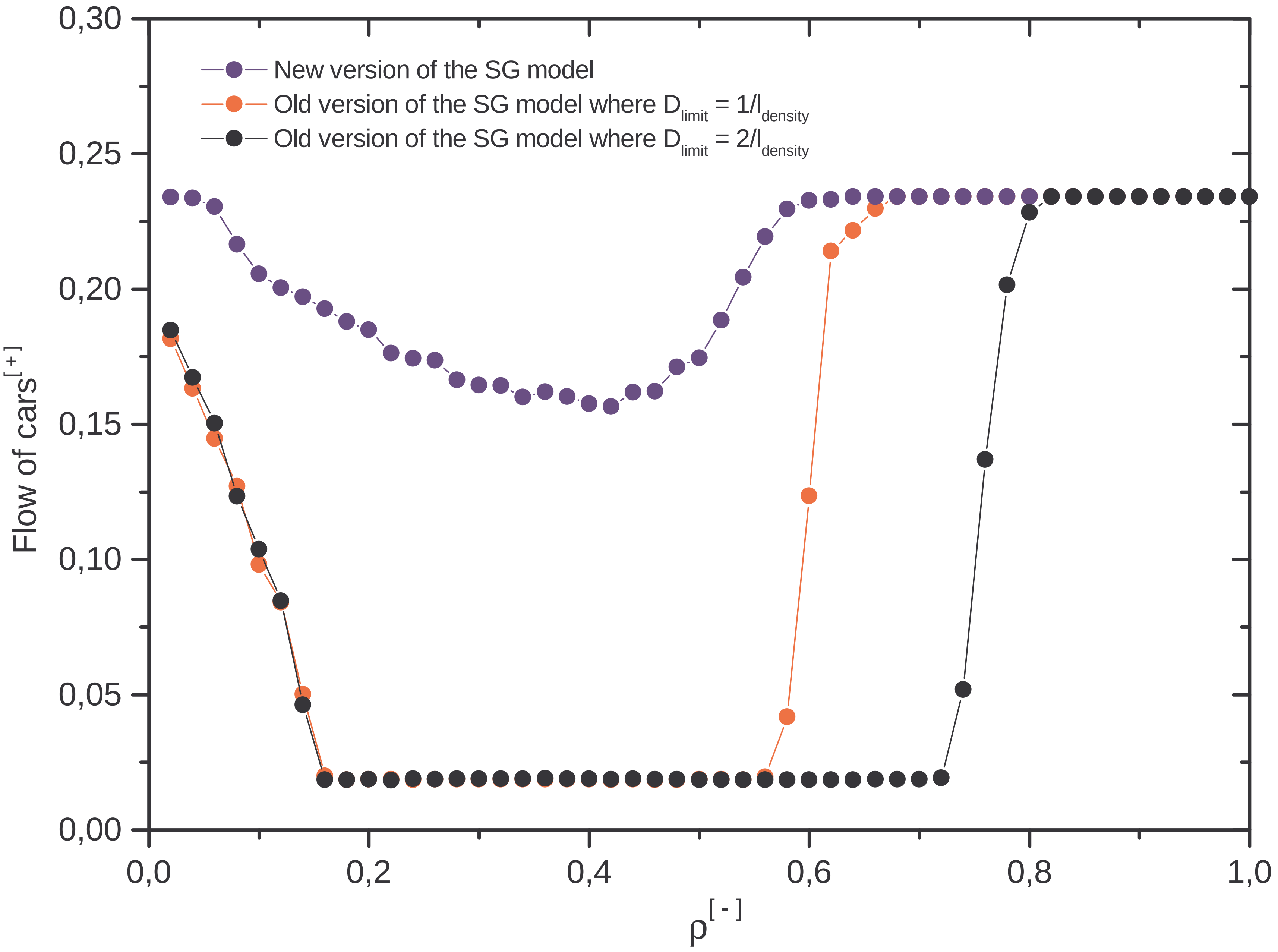
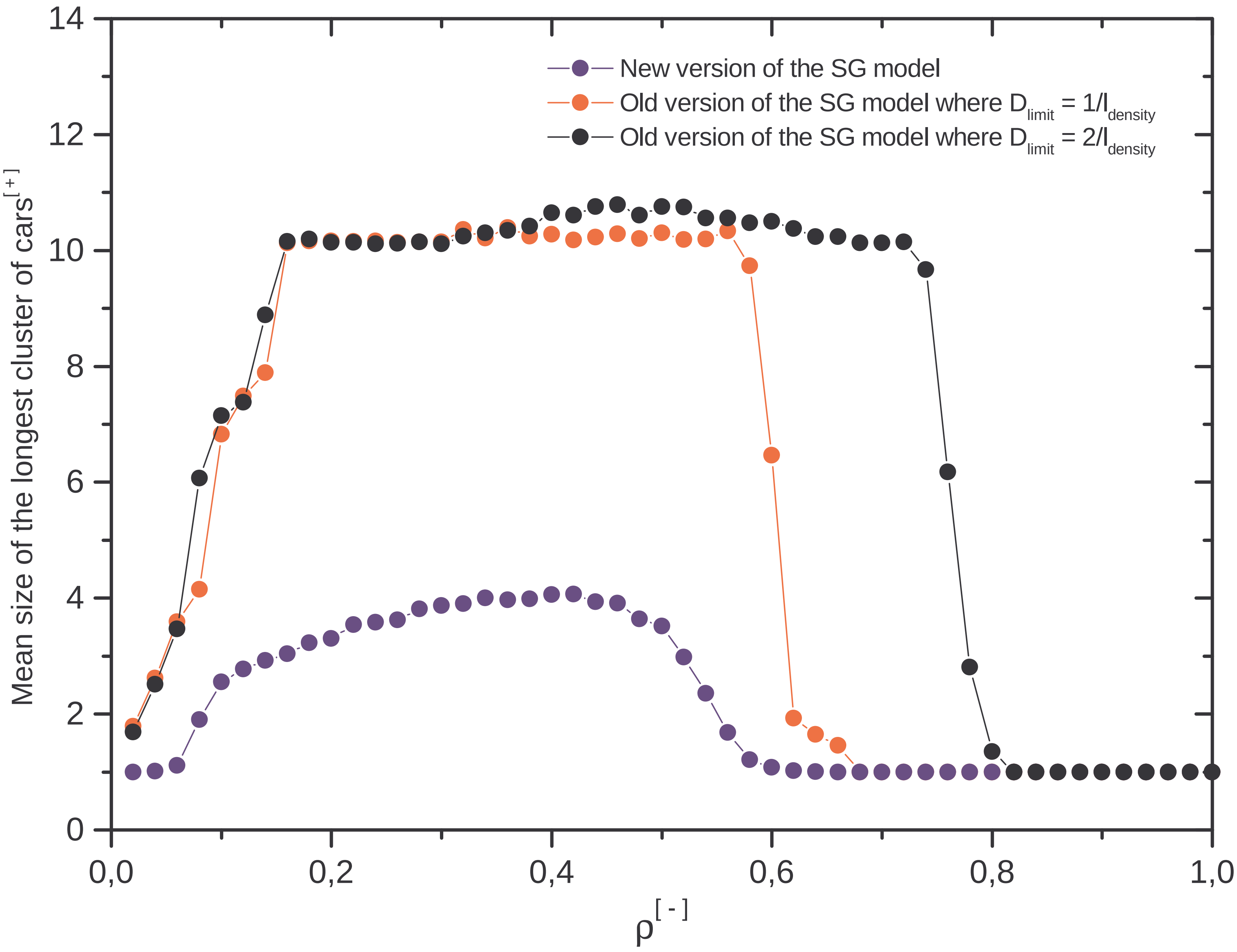


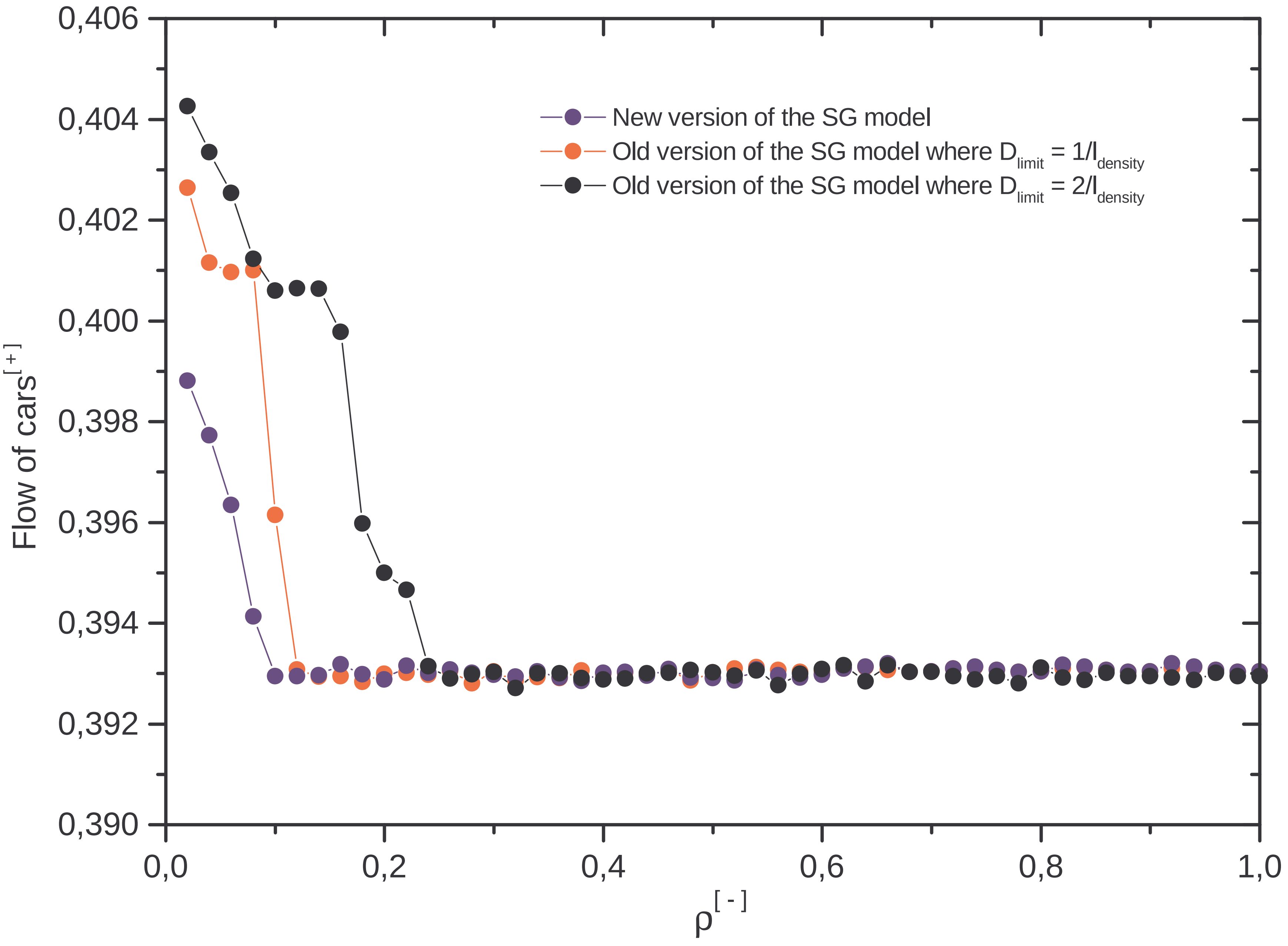
Figure 1b



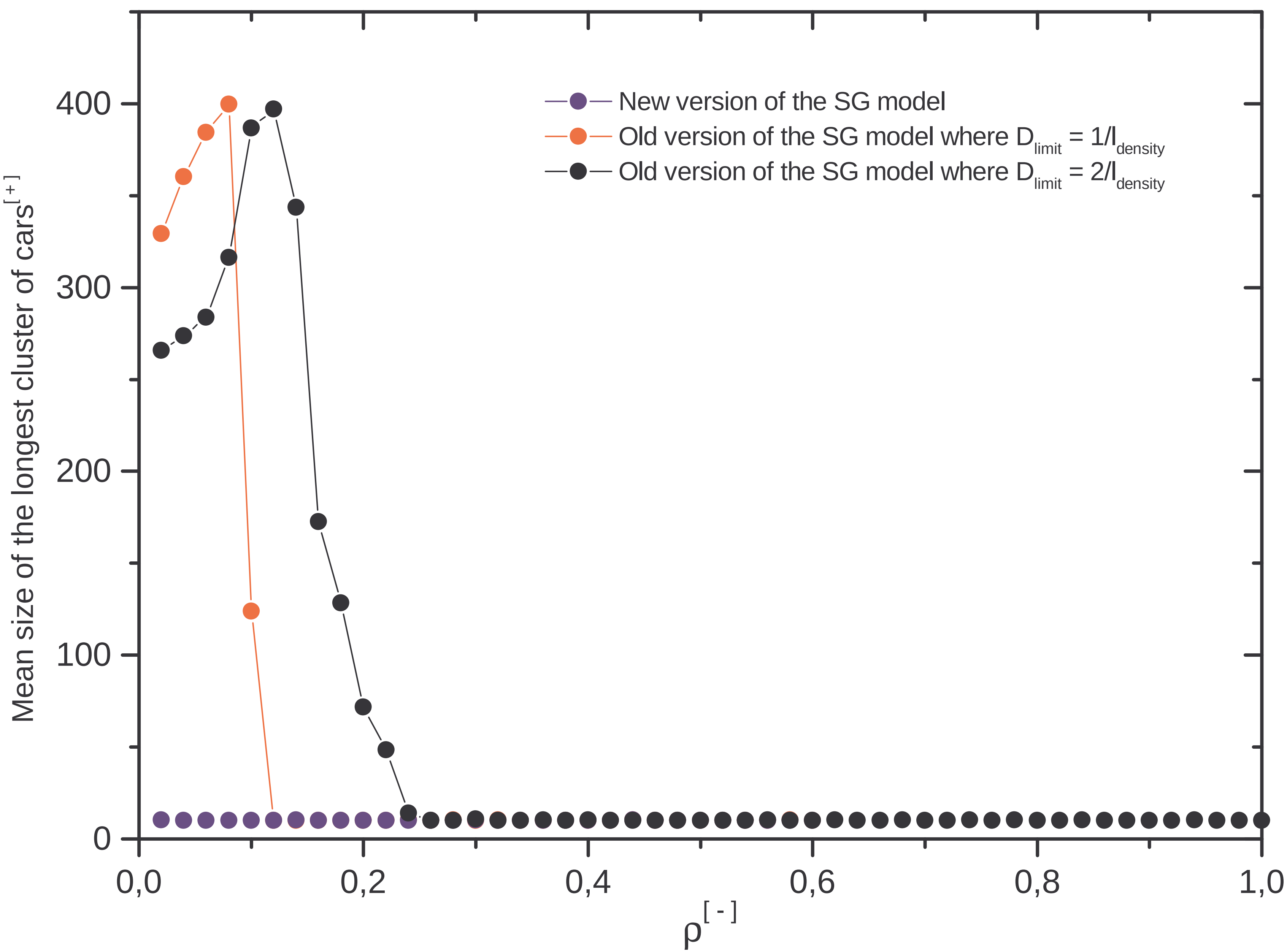
New version of the SG model
Old version of the SG model where D limit = 1/l density
Old version of the SG model where D limit = 2/l density
Flow of cars [+]
ρ [-]
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,0
0,2
0,4
0,6
0,8
1,0



New version of the SG model
Old version of the SG model where D_{limit} = 1/$l_{density}$
Old version of the SG model where D_{limit} = 2/$l_{density}$
Mean size of the longest cluster of cars[+]
ρ[-]
0
2
4
6
8
10
12
14
0,0
0,2
0,4
0,6
0,8
1,0



New version of the SG model
Old version of the SG model where $D_{limit} = 1/l_{density}$
Old version of the SG model where $D_{limit} = 2/l_{density}$
Flow of cars$^{[+]}$
$\rho^{[-]}$
0,406
0,404
0,402
0,400
0,398
0,396
0,394
0,392
0,390
0,0
0,2
0,4
0,6
0,8
1,0


New version of the SG model
Old version of the SG model where D_{limit} = $1/l_{density}$
Old version of the SG model where D_{limit} = $2/l_{density}$
Mean size of the longest cluster of cars[+]
400
300
200
100
0
0,0
0,2
0,4
0,6
0,8
1,0
ρ[-]